Diploma / Research / Semester Thesis

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Description. A simple polygonization is a planar straight-line drawing of a Hamiltonian circuit whose vertices are given points embedded on the plane. In other words, it connects all the vertices with straight lines into a cycle such that the induced polygon has no self-crossing.

Here comes the question, given the number of points, but the arrangement is not fixed, how many different simple polygonizations can be admitted? More formally, let n be the size of the point set and $P(n)$ be the number of simple polygonizations, what is the upper bound of $P(n)$ in terms of n?

Whether $P(n)$ has an analytical expression or not still remains open, but it is believed to have an exponential asymptotic b^n for some constant b. The best lower bound up to now is 4.64^n in [2], achieved by an explicit configuration of the points. The best upper bound is 54.55^n in [4], which is a direct implication of the upper bound of non-crossing perfect matching.

Besides, simple polygonization is also proven closely related to some other problems in combinatorial geometry, likewise triangulation [1], surrounding polygon [5], non-crossing spanning trees [3] and non-crossing perfect matching [4].

Goal. The primary goal of this project is to derive an improved estimation of $P(n)$. Meanwhile, we also have a keen interest in practice-inclined applications and other related topics. For example, propose an enumeration algorithm for these geometry structures (polygonization, non-crossing matching, and so on), or find a construction algorithm for them.

Prerequisite. Interests in combinatorics and geometry. No specific background knowledge is required.

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References

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[3] Naoki Katoh and Shin-ichi Tanigawa. Enumerating edge-constrained triangulations and edge-constrained non-crossing geometric spanning trees. *Discrete applied mathematics*, 157(17):3569–3585, 2009.

[4] Micha Sharir, Adam Sheffer, and Emo Welzl. Counting plane graphs: perfect matchings, spanning cycles, and kasteleyn's technique. In *Proceedings of the twenty-eighth annual symposium on Computational geometry*, pages 189–198, 2012.

[5] Katsuhisa Yamanaka, David Avis, Takashi Horiyama, Yoshio Okamoto, Ryuhei Uehara, and Tanami Yamauchi. Algorithmic enumeration of surrounding polygons. *Discrete Applied Mathematics*, 303:305–313, 2021.